BFI Working Paper Series
No. 2012-013

The Welfare Policy Dilemma of a Negative Income Tax System: Normative Approach

Joseph E. Mullat

October 19, 2012
The Welfare Policy Dilemma of a Negative Income Tax System: Normative Approach

October 19, 2012

Joseph E. Mullat

Abstract:

The question of whether a welfare policy is just and fair to all is a subject to debates. To find a solution, we address a pertinent issue of Negative Income Tax—NIT System, affected by three key actors partaking in welfare-related negotiations. The first actor/negotiator advocates, in the role of social organizations, for citizens' legal and moral rights for the delivery of primary needs. The second negotiator, in the role of public organizations, negotiates in response to non-primary but vital wants for the delivery of public goods and services. The third—the external player, who represents taxpayers, prefers personal consumption to moral and social understanding. The goal of taxpayers is to reduce through voting taxpayers' liabilities deposited as individual income taxes in the joint account of actors/negotiators. The threat, emanating from political and electoral maneuvering of taxpayers, may place the negotiations at risk of collapse. When the collapse is looming closer, the negotiations may breakdown prematurely. The rules and regulations of the bargaining game, and the conditions for unanimous consent, support the claim that income redistribution marginalized by 50% median income is a realistic solution in terms of NIT System.

JEL: C78, H21

Key words: bargaining, policy, public goods, simulation, taxation, voting

---

An earlier version of this paper was presented at the Third International Conference on Public Economics, PET02, Paris, July 4—6 2002 and at the Conference of Economic Design, SED04, Mallorca, June 29—July 2 2004. An extended but older version of the paper has been deposited into Munich Personal RePEc Archive, 24932.

Independent researcher, former docent at the Faculty of Economics, Tallinn Technical University, Estonia. Current residence: Byvej 269, 2650 Hvidovre, Denmark;mailto: jm@mail-telia.dk. This is an Eastern European academic title. Docent is equivalent to associate professor in USA, but it does not exist in UK.
1. Introduction

As a Welfare State presupposes the existence of both a market economy and a democratic political system, its hallmark is that the government intervenes in the distribution of goods and services in the form of subsidies and benefits. This ensures a more egalitarian allocation of wealth and income1 than can be provided by the free market. The dilemma that welfare policy-makers face comes from whether the allocation is just and fair to all. The solution depends on many factors, including main benefactors by which objectives the allocation should be set. The term "welfare", in this context, is used deliberately since the emphases are nothing but income redistribution.

The rules and regulations of income redistribution are subject to debates that typically focus on what the State "ought to" or "should" not deliver. Wider and more substantial benefits and subsidies could be problematic as they might encourage certain behaviors, such as low savings or productivity when economic security is guaranteed; high wage demands because the State ensures the event of unemployment; low social and geographical mobility due to high payment; black labor market that avoids paying high taxes; "moonlighting"—people working multiple jobs; and finally working and studying "too little" because, for many, it simply does not pay to "run longer and faster," cf., Oakley and Saunders (2011: 14). The effect of too high subsidies might be that the human capital does not develop quickly and well enough. Thus, our primary goal is to forecast a warning for policy-makers about all these long-term destructive effects and consequences of the increase in benefits and subsidies.

In our scheme, the agents with low incomes, below certain level, receive subsidies, whereas those with higher incomes, above the level, do not. In this regard, it should be noted that in 1962 Milton Friedman (2002: 190-195) proposed a similar scheme of income redistribution, combined with a flat tax, called a negative income tax system—abbreviated NIT. By rules of negative income tax system low-paid agent receives a payment proportional to the amount by which his/her earnings fell short of a predetermined NIT-poverty line. Low-paid agent's total payroll—the basic income plus NIT payment—is not entitled for taxation. Binding low-paid agents' total payroll for tax purposes, in accordance with the rules and regulations in force, is more practical. Regardless of whether the payroll is a subject to taxation, these NIT redistribution rules are politically sterile—we continue to be in line with NIT-system. A drawback to the system is the fraud extent among low-paid agents receiving NIT payment. In view of that, we introduce the so-called welfare hazard incentives, later referred to as h-effect.

Using a theoretical model of fictitious political organizations, a masquerade of real life or scenario of realistic utopia will be considered. Two actors are playing a bargaining game trying to enlarge their share of wealth, called the welfare-pie. Public organizations tend to focus on regulating private business, limiting government's use of its powers, to combat criminal violence and commercial fraud. Social organizations tend to see the source of evil in disproportion of consumption, unjust redistribution of wealth and income, the profit motive and private property. Social organizations prefer an equitable sharing of the available stock of goods and services here and now. Both organizations are aware of citizens' tax sacrifices to finance subsidies and services. In accordance with elevated NIT-payments, given the rules and regulations financing subsidies, the quantity of benefits/subsidies to be delivered will be

---

1 We use the term wealth narrowly defined as “prosperity or a commodity” delivered through tax channels, and distributed by the State. There is no universal agreement upon the definition.
greater. Consequently, the tax burden of all citizens is increasing. The tax burden and private budget for consumption always lie at the heart of customers' requirements. These requirements lead to political debates about private consumption and to conflict of interests over tax policies in relation to citizens who—as voters—represent social and public organizations.

We hope that our brief remarks clarified some goals of the State by which we might conclude that welfare policy in a representative democracy always faces conflicting interests of organizations. In addition to our primary goal, the aim of this paper is to shed light on how an organizational consensus is reached and whether the consensus reflects a criterion of tax policy as the least burden for all taxpayers. To address this issue, as previously stated, we use two fictitious actors/negotiators. First, we entitle actors/negotiators with a political mandate to initiate proposals that confirm expectations of citizens for better welfare. Second, citizens' desire for excessive delivery of public goods and services is constrained by balance of payments for benefits/subsidies claimed by citizens. This, in turn, brings the scene of negotiations under control of taxpayers. The negotiators are thus forced to act within the imposed constraints, in the roles of social and public organizations, in order to obtain funding for their proposals. This is the classic problem of social planning and public finance, or the welfare-pie allocation, set up by an alternating-offers bargaining game on how to slice individual income taxes, also called income taxes. The choice of the third actor behind the scene, the representative voter-taxpayers, is thus limited solely to voting for or against higher taxes.

We believe that the increase in public spending, reflected by an increase in the size of the welfare-pie, will depend on progressively of the increase of NIT-poverty line as a parameter. However, with the pie progressive increasing in size, the poverty line parameter will shape the expectations of negotiators in different ways. For example, while moving along the line at x-axis, y-axis in Fig. 1 displays single peaked expectations of social organizations. In comparison, concave expectations of public organizations are shown in Fig. 2. The elevated single peaked curve $S$ in Fig. 1 corresponds to the lower but progressively increasing concave curve $P$ of expectations in Fig. 2, and other work around for $P$ to $P$. In support to our belief, the expectations, shaped in this way, emerge within a two-man economy endowed by agents' income abilities marginalized at the level of NIT-poverty line.

![Figure 1. Social organizations' expectations](image1)

![Figure 2. Public organizations' expectations](image2)
Given these hypothetical expectations, called non-conforming, social organizations, when an offer is made (irrespective of its originator), tend to control the NIT-poverty line parameter \( \xi \) independently. Varying the parameter, social organizations may well reach the peak of their expectations. In making those suppositions, we agree with the statement of Rawls' (1971: 304) precepts of justice: "The sum of transfers and benefits..."..."from essential public goods should be arranged so as to enhance the expectations of the least favored consistent with the required saving and the maintenance of equal liberties."

In our scheme, a fictitious agent’s income, at the level of poverty line, is later referred to as NIT-gross level. Provided the tax sacrifice of such an agent is a progressively increasing function of the gross level, and negotiators in eventual agreement commit to how to slice the pie, then, the agent’s NIT-netto level, when the tax sacrifice is deducted from the gross level, becomes a single peaked function. According to Black (1948: 27), single peaked expectations play a major role in collective decision-making when the decision is reached by a vote.

The players at the bargaining table are trying to split an economic surplus in a rational and efficient way. To be sure, the main problem, what the players try to solve during negotiations, is the slicing of the pie. Slicing depends upon characteristics and expectations of bargainers. Given that the expectations of players are non-conforming, i.e., single peaked for the first in contrast to the other, the traditional bargaining procedure may be put differently—no longer a modus operandi of how to split the pie. The procedure, instead of slices, can be resettled, then, to proceed on distinct levels of one parameter—inside NIT-gross level parametrical interval, which turns to be the scope of negotiations. In fact, Cardona and Ponsatti (2007: 628) noticed that "the bargaining problem is not radically different from negotiations to split a private surplus," when all in the bargain have the same, conforming expectations. This is even true when the expectations of the second player are principally non-conforming, not single peaked but concave. Indeed, in the case of non-conforming expectations, related to individual rationality, (Roth 1977), also known as "well defined bargaining problem" or "bargaining set," the scope of negotiations allows dropping the axiom of "Pareto Efficiency." Thus, combined with the breakdown point—a very low poverty line outside the scope—the well-defined problem, instead of slices, can be solved inside NIT parametrical interval.

With these remarks in mind, the procedures of negotiating on slices and NIT-gross levels can be perceived as two sides of the same bargain portfolio. Therefore, it is irrelevant whether the players are bargaining on slices of the pie, or trying to agree on gross levels. Hereby, the main advantage of parametric procedure is exhibited: it brings about a number of different patterns of interpretations of outcomes in the game, linking an outcome, for example, to the lowest tax rate, the lowest amount of taxable income, etc., all as indicators of taxpayers' most desirable sacrifices. In consideration of alternative ways to establish the NIT-gross level, which describes outcomes of collective bargaining in the form of voting, or voting in any voting scheme, in the form of bargaining—the scope of negotiations brings the voting and bargaining under one roof. In this way, we hope to enrich the range of interpretations for both the bargaining and voting schemes.

In our welfare game, the asymmetric Nash solution incorporates the breakdown point into a power indicator of social organizations. To address the indicator properly, the indicator cannot be given exogenously. To overcome the obstacle, we supply the game with a point of breakdown extracted endogenously on condition referred to as pre-equity of breakdown.

---

2 We say also interpersonally incompatible expectations, impossible to match by a monotone transformation.
Beyond the perception of how to negotiate an expected slice of the welfare-pie, it is also reasonable to believe that income distribution is, perhaps, the only target for control and an exclusive source of information for assessing the welfare policy. There is neither a panel data available, based on credible income redistribution, nor experimental support for the welfare game justification. Even as this key weakness is understood, the solution of the game, simulated by numerical calculations, yields an NIT-gross level sufficiently close to be considered a realistic match (Table 1) to "what amounts to a moving poverty line at 50% of median incomes," cf., Fuchs point, (Bowman 1973: 55). Rawls (1971: 98) introduced Fuchs point as an alternative to the measurement of poverty with no reference to social position.

At present none of the extant literature sources has addressed the welfare issue directly by relating the slicing of the welfare-pie to (a) public finance, (b) alternating-offers game, and (c) income redistribution. We slice the welfare-pie in an unconventional way by applying non-conforming bargaining scheme. Our asymmetric scheme is forward-looking in that it aims to include such a tripod-cascade in the calculus of bargaining game solutions. It might help to search for some ideas in terms of the size of the pie as a parameter, i.e., how to shed light on the production situation. Indeed, "the situation need not be a zero sum game. Tactics, that determine the division can affect the size of the pie," wrote Leibenstein (1979: 493). Clarifying these guidelines, Altman (2006: 149) wrote: "There are two components to the productivity problem: one relates to the determination of the size of the pie, while the second relates to the division of the pie. Looked upon independently, all agents can jointly gain by increasing the pie size, but optimal pie size is determined by the division of pie size."

Therefore, in order to explain the root cause of the results and to find solutions for the welfare game, we try to move in all three directions of the scheme aiming to bring to the surface the economic content in a rigorous mathematical form, and to lay the foundation of the results presented along a cascade of the following modules:

**Fiscal policy**

While on delivery to its end destinations, the welfare-pie (the wealth or income taxes), once balanced fiscally, must remain balanced throughout and in spite of volatility in the economy;

**Bargaining**

The negotiations between social and public organizations on how to slice the welfare pie comply with the rules and regulations in the alternating-offers bargaining game;

**Unanimous consent**

Bringing a motion to a vote is necessary to meet consumer perception against high taxes and excessive public spending. Whether it is good or bad or whether it ought to be acknowledged or not, or rejected or accepted, this motion must be carried out by the unanimous consent of voter-citizens.

The tripod scheme can be understood as requirements to be met by rules and regulations of the welfare game, cf. "Rational man" deliberation, (Rubinstein 1998: 7). We aim to show that the scheme will enable us to affirm the view under which these modules are embedded as the cascade into welfare policy of the State. Hence, in evaluating income redistribution policies, poverty and income distribution analysis can be subject to and be performed by computer simulations. This initiative could also serve to unify the theoretical structure of economic analysis of institutions, for passing judgment on social and public organizations, or for systematic inquiry into impacts of governmental decisions and actions.

In Section 2, we invite the reader to play a simple game in a way that illustrates the standard of how to adjust the bargaining power of negotiators to ensure agreement if a specific outcome of negotiations or social planning of organizations is desirable. We also hope that, before advancing any further, the reader will try to solve our pre-exercise at the beginning of this section. Before delving deeper into the analysis, we discuss the assumptions to be made a priori based on the expectations of players involved. In the following three sections, we present the modules of our tripod-cascade. In Section 3, we disclose fiscally safe welfare
The policy of social organizations in amalgamation with a static balance for funding subsidies. Referred to as volatility constraint, the amalgamation dynamically holds down an inverse working incentives phenomenon—the welfare hazard effect. In Section 4, the citizens' ambivalence and multifaceted welfare perception is discussed from the angle of the game of alternating-offers. Our work here associates the policy on poverty with bargaining related to monetary expectations of two actors—the social and public organizations. Thus, in principle, given arbitrary income distribution, it would be possible, within the scope of negotiations, to obtain an exact analytical solution of the game.

While consolidating a draft to the agreement, the actors on the opposite sides of the bargaining table might disagree over the terms of outcomes and delay the decision. As a consequence, the draft might not necessarily yield the best outcome for citizens. In response, the citizens, in the role of voters, may emanate by political and electoral maneuvering a threat to vote against the draft, bringing the negotiations at risk of premature collapse, or walking away from the bargaining table. Thus, in accord with bargaining procedure, we reveal in Section 5 an appropriately settled bargaining problem for the game that probably enables voter-citizens to accept an outcome by unanimous consent. In Section 6, we discuss a pre-equity condition of endogenously determining a breakdown point of the game. The findings of the study are presented in Section 7, and Section 8 summarizes this work with a postscript, followed by rules and regulations associated with the game. Calculus is illustrated separately.

2. Preliminaries

Welfare economists believe that just and fair income redistribution is achieved through efficient allocation of society's resources. Since the State has a duty to help the less fortunate, we refer to income redistribution in a two-fold manner: redistribution of basic necessities or primary goods, like shelter, clean and fresh water, nutrition, etc., and of non-primary goods, like, national defense, public-safety and order, roads and highway systems, etc. Thus, while identifying the basis for welfare policy, we must find the point for efficient allocation of primary and non-primary goods. Fundamentally, the income redistribution—the welfare-pie slicing—is about just and fair delivery of both goods, preserved traditionally as public goods. In our variety of names, we put different "shrinking labels" on public goods as non-primary but vital goods. Incidentally, when primary and non-primary goods are on delivery to their end destinations, we preserve the conventional nomenclature for both goods as public goods.

Efficient allocation of society's resources implies that a consensus among political organizations about just and fair income redistribution of primary and non-primary goods might be reachable. Despite prevailing views to the contrary, we hope that the process of finding the solution to the game with the welfare-pie is the best way to understand the dynamics of this process. Particularly, in our view, greater demands of consumption of basic goods lead to the deterioration of welfare quality of all citizens, including the less fortunate themselves. Therefore, we first invite the reader to play the game, which explains the situation in simple terms.

2.1 Simple game. The game demonstrates how a welfare-pie is fairly sliced between negotiators. Social organizations—(SO) is a soft negotiator, not very keen on public, non-primary goods but with emphasis on basic, primary goods. Given the vital nature of non-primary goods, the public organizations—(PO) is a tough negotiator and prefers public goods.

3 Note that for the purpose of the game we do not ignore the size of the pie but put this issue aside temporarily until the next subsection.
The axiomatic bargaining theory finds the asymmetric Nash solution by maximizing the product of players' expectations above the disagreement point $d = \langle d_1, d_2 \rangle$:

$$\arg \max_{0 \leq x, y \leq 1} f(x, y, \alpha) = (u(x) - d_1)^\alpha \cdot (g(y) - d_2)^{1-\alpha},$$

the asymmetric variant (Kalai, 1977).

Although the answer may be known to game theory purists, the questions often asked by many include: What are $x$, $y$, $\alpha$, $u(x)$ and $g(y)$? What does the point $\langle d_1, d_2 \rangle$ mean? How the arg max formula is used? The simple answer can be given as:

- $x$ is SO's slicing the pie, and $\alpha$ is SO's bargaining power, $0 \leq x \leq 1$, $0 \leq \alpha \leq 1$; $u(x)$ is SO's expectation, for example $u(x) \equiv x$, of SO's $x$ slicing the pie;
- $y$ is PO's slicing the pie, and $1-\alpha$ is PO's bargaining power, $0 \leq y \leq 1$; $g(y)$ is PO's expectation, for example $g(y) \equiv \sqrt{y}$, of PO's $y$ slicing the pie.

Based on the widely accepted nomenclature, we call $s = \langle u(x), g(y) \rangle$ the utility pair. The disagreement point $d = \langle d_1, d_2 \rangle$ is what SO and PO collect if they disagree on how to slice the pie. The welfare-pie disagreement point is $d = \langle d_1, d_2 \rangle = (0, 0)$, whereby the players collect nothing. Further, we believe that expectations from the pie are more valuable for PO, indicating PO's desire $g(\frac{y}{x}) = \sqrt{\frac{y}{x}} = 0.707$, which is greater than SO's desire $u(\frac{x}{y}) = 0.5$.

Now considering the arg max formula of $f(x, y, \alpha)$, one may ask a new question: What is the standard that will assist the social planning of government to redesign bargaining power $\alpha$ of social organizations facilitating SO during negotiations to obtain a desired half of the pie? Public organizations may only accept or reject the proposal. A technical person can shed light on the solution. We can start by replacing $u(x)$ with $x$, $y = 1-x$, $g(y)$ with $\sqrt{1-x}$, and taking the derivative of the result $f(x, 1-x, \alpha)$ with respect to the variable $x$ by evaluating $f'_x(x, 1-x, \alpha)$. Finally, with $x = \frac{y}{x}$, the equation $f'_x(\frac{y}{x}, \frac{y}{x}, \alpha) = 0$ can be solved for $\alpha$; indeed the equation $f'_x(\frac{1}{2}, \frac{1}{2}, \alpha) = 0$ resolves for $\alpha = 1/3$.

In general, one might feel comfort in the following judgment: "Even in the face of the fact that PO are twice as tough a negotiator\(^4\) to count on the half of the pie is a realistic attitude toward SO's position of negotiations. Surely, rather sooner than later, since SO revealed that PO prefers public goods as vital goods whatever they are, SO would have PO agree to a concession." This attitude might well be the standard of social planning in redesigning the power of organizations if half of the pie is desirable as a specific outcome of negotiations.

2.2 Pre-exercise. Next, it will be assumed that in the background of SO's judgment, the quality of the pie first increases, when the size is small, but reaching the peak point it starts to decline: the quality, one will say, is single peaked upon the size. For PO, the pie is always desirable. The above can be restated, then, with the condition that SO seeks an efficient size $z$ of the pie. Although SO is not committed to size $z$ whether or not to accept the recommendation of PO, it is committed, however, to slice $x$ aligned in eventual agreement. Let the utility pair $\langle u, g \rangle$ of SO's and PO's expectations be given by:

$$u(z, x) = z \cdot \left(\frac{1}{2} + \frac{x}{2} - z\right), \quad g(z, y) = z \cdot \sqrt{y}, \quad z \in [0, 1], \quad x, y \in [0, 1].$$

\(^4\) Let us say that PO pay for political efforts to get its share of the pie to be twice as much of that of social organizations.
Define efficient slices to the size $z$ as a curve $x(z)$, which solves $u'_z(z,x) = 0$ for $x$. Evaluating $x$ from $u'_z(z,x) = 0$ and subsequently replacing $x(z)$ into $u(z,x)$ and $g(z,x)$, yields $u(z) = z^2$ and $g(z) = z \cdot \sqrt{3 - 4 \cdot z}$. Hereby, the bargaining problem $\langle S, d \rangle$ reschedules, then, into parametric space $S = \langle u(z), g(z) \rangle$ of the size parameter $z \in [\frac{1}{2}, \frac{3}{4}] \subseteq [0,1]$. Given the scope of negotiations—the interval $[\frac{1}{2}, \frac{3}{4}]$—the root $\frac{1}{2}$ resolves $u'_z(z_{\frac{1}{2}}) = 0$, and the root $\frac{3}{4}$ resolves $u'_z(z_{\frac{3}{4}}) = 0$ for $z$ accordingly. In SO's view the pie must fit the size, since outside the interval $[\frac{1}{2}, \frac{3}{4}]$ it is too small—not useful at all, or too large—and offers a low quality of welfare services. Therefore, the disagreement occurs at $d = \langle u(\frac{1}{2}), g(\frac{3}{4}) \rangle = \langle \frac{3}{4}, 0 \rangle$. Assuming that the size of the welfare-pie remained fixed (fiscally safe) during the delivery to its end destinations, the Nash symmetric solution to the game is found at $z = 0.69$, $x = 0.74$. On the other hand, SO's asymmetric power $0.212$ is adequate to negotiate with PO about the half of the pie. The size $z = 0.62$, supplemented by power $0.212$, fits the capacities of an economy for income redistribution.

2.3 Settings. Having discussed the reasons for slicing the public goods into basic and other wants, it is deemed reasonable to follow the same pattern in reality, whereby social organizations negotiate favorable portions, i.e., the slices $x$, $0 \leq x \leq 1$, of the pie with public organizations. Following the traditional rules of the alternating-offers game of how to slice the welfare-pie, when the pie is desirable for both sides, the negotiators (bargainers), changing roles, commit to offers $(x, y)$, $x + y = 1$. Thus, establishing the rules and regulations of social organizations, valid for financing a desirable level of subsidies, requires an NIT parameter $\xi$. In contrast to our simple game, such a requirement is a separate matter connected to the size of the pie. To finance subsidies, the size of the pie (the wealth) can be accounted for by assuming that higher values of the NIT-gross level $\xi$ need an increased marginal tax rate $\tau(\sigma, x)$. Therefore, while increasing the wealth through tax channels, we suppose a positive acceleration $\tau''(\sigma, x) > 0$ of the rate $\tau(\sigma, x)$, $\tau'(\sigma, x) > 0$ for all agents, inclusive $\sigma = \xi$, $\sigma$ is an income of agent named $\sigma$.

We assume that a proportion $r$ of $(\xi - \sigma)$ compensates for the unfair subsistence of the less fortunate agent $\sigma < \xi$ and is a monetary compensation for the agent to buy an eligible "poverty basket" of food, clothing and shelter, fuel and lights, etc. According to Rawls' (1971: 92) view that "primary goods are things which it is supposed a rational man wants whatever he wants," we define the expectation of social organizations by a function $u(\xi, x)$; $u(\xi, x)$ is the after tax rank of an agent with income equal to NIT-gross level $\xi$, also named further on by NIT-netto level. All except expenses on subsidies represent the expectation.

5 Once again, to find the Nash symmetric solution a technically minded person must resolve the equation $f'_z(z, \alpha) = 0$ for $z$, where $f(z, \alpha) = (u(z) - \frac{3}{4}) \cdot g(z)^{1\alpha}$ when $\alpha = \frac{1}{2}$; $z = 0.69$ resolves the equation. Then, resolve the equation $u'_z(0.69, x) = 0$ for $x$, and find that $x = 0.74$. To find the power of asymmetric solution, first resolve the equation $u'_z(z, \frac{1}{2}) = 0$ for $z$, $z = 0.62$, $x = \frac{3}{4}$. Then, resolve $f'_z(0.62, \alpha) = 0$ for $\alpha$ and find that the power of SA equivalent to $\alpha = 0.212$ is suitable to negotiate with PA when the equal, $\frac{1}{2}$-slicing of the pie is desirable.
\( g(\xi, x) \) of public organizations, named above as "public goods." With the proviso that negotiators commit to the slice \( x \), we can further assume that function \( u \) is a single \( \cap \)-peaked upon \( \xi \) increase. Expectations \( g \) of public organizations are decreasing with \( x \), but increasing with \( \xi \). Both expectations \( \langle u, g \rangle \), cf. Fig. 1-2, are considered analytic functions \( u(\xi, x), g(\xi, x) \), described as follows. Given the previously mentioned interval \([\xi_1 \leq \xi \leq \xi_2]\), later referred to as the scope of negotiations, \( u \) is supposed to be single \( \cap \)-peaked, \( u''(\xi_1, x) < 0 \) upon \( \xi \) increase, \( u'_i(\xi_2, x) > 0 \), \( u''_i(\xi_2, x) < 0 \). In addition, upon \( x \) increase, utilities \( u \) are convex, \( u'' > 0 \), \( u' > 0 \), whereas following the increase in \( \xi \), utilities \( g \) should be concave with \( g'' > 0 \), \( g'' > 0 \). With \( x \) increase, utilities \( g \) always decrease, i.e., in both cases, either \( g \) is convex, \( g'' > 0 \), or concave, \( g'' < 0 \).

3. Fiscally safe policies of social organizations

The delivery of primary goods, which counteracts negative contingency, if it occurs, is the main responsibility of the social organizations. Social organizations' intervention is universal in the sense that it covers the entire population, regardless of one's situation before or after the contingency. If primary goods are of sufficiently high quality (Greve, 2008: 58), poverty is not allowed. This of course provides a relatively high level of social spending and high level of taxes, creating adverse conditions of fiscally unsafe budget in balance of payments on social welfare, i.e., causing a misbalance in income taxes. Hence, a high level of primary goods, secured largely independently of market forces, might have a conflict-driven effect on welfare policy, which should not be borne by citizens alone, since, as already noted, the State has a duty to help those down on their luck.

Following the conflict-driven simple game analogy, when political organizations were trying to sign an agreement, first actor looked for an efficient size of the welfare-pie. In doing so, the first actor prescribed the size and proposed a slice of pie, which the second actor accepted or rejected. When rejected, the second actor proposed the slice but had only an authority to recommend a size that the first actor might not be obligated to accept. We also supposed that upon delivery to its end destinations the welfare-pie remained fiscally safe. By these rules of alternating offers procedure the game continued until, at the end, within the scope of negotiations, the agreement was reached. The first actor under any such an agreement had a commitment to the slice without a commitment to the size.

Assume now, on the contrary, while public spending rises, both sides know that upon delivery the size of the pie might change. On the backdrop of changing size, the changes lead to misbalance of subsidies and income taxes. Misbalance of subsidies can put the size in doubt or make it even fuzzier! This difficulty may drive the fair bargain to unknown destiny, delaying decisions. Under such circumstances, the welfare-pie is not fiscally safe anymore, and may affect the behavior of our rational negotiators. Therefore, a fiscally safe plan of slicing and delivery of the pie of taxes is needed. Otherwise, unless the welfare policies do not enforce fiscal safety, the rules and regulations of the game are not living up to their claims. All that is needed is a criterion of how to detect a fiscally safe welfare policy.
In our further discussions, quantification is used as a measurement to address this issue. We take a quantum of average income for the measurement of all variables and functions over income distribution samples \( P(\sigma, \xi) \) parameterized by NIT-gross level \( \xi \). Hence, the average income per capita establishes the ratio scale.

It is helpful to focus first on welfare policy without any warranty of fiscal safety. The policy might be set by the NIT-gross level \( \xi \) to decide who needs the transfer of wealth from the wealthy. Thus, the variable \( \sigma \) to the left from the parameter \( \xi \), \( 0 < \sigma \leq \xi \), allocates the income of the needy or the benefit claimant; the income of the wealthy to the right is \( \sigma \), \( \xi < \sigma < \infty \). A benefit claimant \( \sigma < \xi \) claims and receives a benefit or subsidy proportional to \( \xi - \sigma \), i.e., \( r \cdot (\xi - \sigma) \), see above. In contrast, agents \( \sigma > \xi \), as well as agents \( \sigma = \xi \) —all those more wealthy agents receive a zero subsidy.

Following, we study a specific scheme emphasizing the readiness of the society for funding public spending. We suppose that average cost \( B \) of subsidies and average taxable income \( W \) depend on NIT parameter \( \xi \), \( B \equiv B(\xi) \), \( W \equiv W(\xi) \) (realistic form in the appendix). As previously narrowly defined, \( W(\xi) \) can refer to the wealth amount. Based on our perception of incomes distribution samples \( P(\sigma, \xi) \) the product \( \tau \cdot W(\xi) \) estimates the average income taxes. Let the average cost of public goods be \( g(\xi) \). Assuming that public spending has reached its end, whereas \( \tau \cdot W(\xi) = B(\xi) + g(\xi) \), the public spending equals the size \( z(\xi) = \tau \cdot W(\xi) \) of the pie. This suggests that the deliveries of primary and non-primary, i.e., the basic and public goods, have reached their destination; the wealth collected through tax channels is spent.

Suppose that negotiators in the welfare game prefer to commit to the slice \( x \) and will agree to hold the balanced way of financing subsidies \( B \), i.e., \( B(\xi) = \tau \cdot x \cdot W(\xi) \). Let the social organizations be ready to finance subsidies, i.e., to refund \( B(\xi) \) via income taxes \( \tau \cdot W(\xi) \). The organizations commit to keep the balance \( B(\xi) = x \cdot \tau \cdot W(\xi) \) of payments between credits \( B(\xi) \) and debts \( x \cdot \tau \cdot W(\xi) \) as a portion \( x \) of income taxes \( \tau \cdot W(\xi) \). The balance specifies welfare policy \( \xi \), an implementation of NIT-gross level \( \xi \), welfare reform, pact, program, etc. Although this balance is valid, it might break in the future putting once or repeatedly the adjustment in \( \xi \) on the agenda; the policy \( \xi \) might turn out to be fiscally unsafe. Given the balance of payments for funding subsidies as a duty, only a few would question the duty; yet, almost all, perhaps for different reasons, prefer a fiscally safe policy to implement the balance. Our focus area, next, will be on a constraint that embodies the fiscal safety of welfare policy \( \xi \) —a criterion for a safe delivery of the pie to its end destinations.

\[ \text{Monetary scale satisfies an interpersonal comparability of utilities (cf. Narens and Luce 1983: 249).} \]
Idempotent rules. The delivery of basic (primary) and public (non-primary) goods does not necessarily safeguard an adequate funding of the expenses; usually the rules of taxation neither match nor prevent numerous changes and adjustments. The problem was already mentioned above: The size of the pie could change too rapidly, requiring frequent adjustments of taxes. We came, thus, upon a sequence \( \xi',\xi'', \ldots \) of multiple adjustments of NIT parameter \( \xi \). The study emphasizes that on delivery, the adjustments of the pie are unnecessary. The size of the pie must be fiscally safe. Otherwise stated, when old decision \( \xi' \) was implemented, the new one \( \xi'' \) taken by the same protocol must coincide with the old. Such schemes, known as idempotent rules, originate from social choice mechanisms. This suggests that multiple adjustments of rules and regulations should not change the machinery of how the subsidies are legally paid out, and, in particular, implemented twice give the same result. Such an understanding requires that, to guarantee the fiscal safety of NIT-gross level, the levels must coincide amid sequence of pairs \( (\xi',\xi'') \) at some matching point \( \xi' = \xi'' \).

In spite of volatility of basic goods \( B(\xi) \) on delivery, the balance for subsidies \( B(\xi) = x \cdot \tau \cdot W(\xi) \) can be seen, also, as immunity of welfare economy. That is the immutability holds down the welfare hazard effect of income redistribution. Given the preceding idempotent scheme under immune or fiscally safe composition \( [B(\xi),W(\xi)] \), this insurance is equivalent to implementing the policy \( \xi \) only once. For this reason, by ensuring the fiscal safety of the balance, we assume that the rules and regulations of the game have been socially planned and redesigned accordingly.

In this mode, the rules and regulations reflect the policy \( \xi \) that outlines the fiscal safety of public spending by which the policy might be brought to conclusion. We therefore conclude that the account on expenses \( x \cdot \tau \cdot W(\xi) \) meant for social spending must be in balance not only for funding subsidies \( B(\xi) \), when the particular policy \( \xi \) takes effect, but also the policy \( \xi \) must enforce a fiscal safety in the full spectrum of current and future events.

Clearly enough, the balance \( B(\xi) = x \cdot \tau \cdot W(\xi) \) is a static relationship leading to functional dependency \( \tau \equiv \frac{B(\xi)}{x \cdot W(\xi)} \) binding \( \xi \) and \( x \) variables. Hereby, the tax rate \( \tau \) becomes a function of \( \xi \) and \( x \), \( \tau \equiv \tau(\xi,x) \). According to Malcomson (1986: 266), the personal allowance parameter \( \phi \) by the tax bracket \( [\phi,\infty) \), establishes agent’s \( \sigma = \xi \) after tax rank \( \pi(\xi,\tau) = (1-\tau) \cdot (\xi - \phi) + \phi \). Thus, the dependency \( \tau \equiv \tau(\xi,x) \) transforms \( \pi(\xi,\tau) \) into realistic NIT-netto level \( \pi(\xi,\tau(\xi,x)) \), or a fiscally safe after tax rank of a fictitious agent. This netto level \( \pi(\xi,\tau(\xi,x)) \) highlights fiscal capabilities of social organizations. Whatever the expenses of basic goods are at the moment, the real cost of living does not necessarily match the NIT-netto level \( \pi(\xi,\tau(\xi,x)) \) ! Therefore, in order to ensure realistic and fiscally safe rules avoiding the welfare hazard effect of this undesirable mismatch, an equation for a fiscally safe NIT-gross level \( \xi \) should be solved, as follows.

---

7 A choice operation (or decision) applied multiple times is idempotent if beyond the initial application it gives the same result, cf., Malishevski (1998: 422). For example, if water is boiled twice, it is still boiled water.
Observation 1. Constraint \( u = \pi(\xi, \tau(\xi, x)) \) is the necessary condition to uphold a fiscal safety of the static balance \( B(\xi) = x \cdot \tau \cdot W(\xi) \).

The observation states, whatever tax increase is implemented, the NIT-netto level \( u \) is too high to be reached when the condition has been violated. All proofs are available on request.

Corollary. Adjustments \( \xi', \xi'', \ldots \) are unnecessary if \( u = \pi(\xi, \tau(\xi, x)) \) is to be solved for \( \xi \). With incomes \( \sigma < \xi \) or \( \sigma > \xi \), the only remaining option to change their social positions is irrational for agents \( \sigma \), and, thus, the root \( \xi \) allows restraint of welfare hazard effect.

Fiscally safe policy \( \xi \) induces fiscally safe composition \([B(\xi), W(\xi)]\), which is the basis for the welfare game solutions. A reasonable question then emerges: Which taxable income \( W(\xi) \) represents fiscally safe welfare policy \( \xi \) for the delivery of subsidies \( B(\xi) \)? The answer is in the following three constraints:

\[ \tau \cdot W(\xi) = B(\xi) + g \]  \hspace{1cm} (1)

\[ B(\xi) = x \cdot \tau \cdot W(\xi) \]  \hspace{1cm} (2)

\[ u = (1 - \tau) \cdot (\xi - \phi) + \phi \]  \hspace{1cm} (3)

Delivery constraint, all taxes are spent, i.e., the delivery of basic and public goods reached its end; this form of constraint makes sense only for proportional, or flat taxes. The case will later substantially simplify the method of function minimization with constraints.

A static balance for funding subsidies with the portion \( x \) of the welfare-pie (income taxes) credited to and deposited (debited) in social organizations' account \( B(\xi) \); it equals to the average of subsidies shifted by the policy \( \xi \).

The constraint confirming delivery of subsidies sets up the fiscal safety of (2). In contrast to \((\sigma, \tau) \in \mathbb{R}^2\), we distinguish NIT-netto levels \( u = \pi(\xi, \tau) \) as indifference curves \((\xi, \tau) \in \mathbb{R} \subset \mathbb{R}^2\)!

Taking the expression \( \tau(\xi, x) = \frac{B(\xi)}{x \cdot W(\xi)} \) out from constraint (2) and replacing \( \frac{B(\xi)}{x \cdot W(\xi)} \) into \( u = \pi(\xi, \tau(\xi, x)) \), constraint (3) must resolve but fiscally safe policy for \( \xi \):

\[ L(\xi, x, u) := (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0 \]  \hspace{1cm} (4)

Constraint (4), also called volatility constraint, sets up the fiscal safety module. It holds down the welfare hazard effect by balance (2) in amalgamation with constraint (3).

\[ \text{Below we continue to call the average taxable income— the wealth.} \]
**Summary.** An outcome \(\phi, \xi \Rightarrow z, x, \alpha, \tau, \langle u, g \rangle\) constitutes the citizens' bargaining shield for income redistribution that relates to a bundle of variables or constants: \(\phi, \xi\) — control, and \(z, x, \alpha, \tau\) — are status variables; \(\langle u, g \rangle\) — players’ competing expectations (proposals):

- \(\phi\) — the personal allowance establishing tax bracket \([\phi, \infty)\);
  it is an ex-ante, a control (tuning) variable, \(0 < \phi = \text{const} < \xi\);
- \(\xi\) — the NIT-gross level; a policy to decide who is living in poverty,
  the choice or the control parameter as well;
- \(z\) — the size \(z = \tau \cdot W(\xi)\) of welfare-pie; the account of income taxes
  that equals public spending per capita in case of proportional taxes;
- \(x\) — the slice of welfare-pie \(z\); a portion \(x\) of \(z\) to be deposited in
  favor of social organizations for funding subsidies, \(0 \leq x \leq 1\);
- \(\alpha\) — the negotiating power of social organizations, \(0 \leq \alpha \leq 1\);
- \(\tau\) — the marginal tax rate, the wealth-tax \(\tau(\xi, x)\) of wealth amount
  \(W(\xi)\) is set up by (1);
- \(u\) — the NIT-netto level, the after tax rank of a fictitious agent with gross income \(\xi\),
  expectation function \(u(\xi, x)\) of social organizations, which is set up by (2) and (3);
- \(g\) — the expectation function \(g(\xi, x)\) of public organizations, which is set up
  by (1) and (2); the account for refund of public goods expenses per capita.

The slice \(x\) and the marginal tax rate \(\tau\), due to constraints 1-3, became functions of
variables \(\xi, g\): \(x \equiv x(\xi, g)\) and \(\tau \equiv \tau(\xi, x(\xi, g))\). This form of dependence appears later
at the module of welfare-pie bargaining.

**4. The Welfare Game of Alternating-Offers**

Suppose that negotiators agreed to play the bargaining game of alternating offers, resulting
in some commitments of how to slice the welfare-pie fairly. Playing the game, rational nego-
tiators have a reason to align the procedure to participate in any eventual agreement. The risk
of premature collapse during negotiations may be the driving force to reach the agreement
sooner rather than later. Having reached a preliminary agreement a new real life problem
arises: Who should make the decision about the size of welfare-pie? Usually players negoti-
ate on such matters when there are equal and symmetric preconditions in place that guarantee
their equal rights. The decision upon the size will thus be an equal obligation of both sides.
Given social skills and competency shortage of public organizations, what we suppose is the
case, the principal-agent problem may arise—who acts on whose behalf? In the spirit of all
these guidelines, it will be reasonable, then, to reduce PO’s scope of mutual duties, while
making welfare decisions—retaining, however, PO’s advisory rights. We proceed as follows.

---

9 Status and Control variables are the prerogatives of control theory.
4.1 Asymmetry. Factors, such as economic growth, decline or stagnation, demographic shift, pit or migration, political change or change in scarcity of resources, property rights, skills and education of the labor force, etc., might turn the rules and regulations for transfers of subsidies and benefits into fiscal inconsistency of a desirable welfare system. The result is that the size of the welfare-pie might be too large or too small, not worth the effort to slice the pie at all. To handle the situation, we assume that SO possesses all relevant skills of welfare policy implied. If the policy does not meet its goal, SO can resort to income redistribution in situations where it can enforce decisions to deliver the wealth properly, or effectively retaliate for breaches—welfare rights movements, protests, civil disobedience, etc. Being short of social skills and competence, if political interests of both sides in the final agreement are different, PO cannot fully control SO actions and intentions. Thus, from SO's viewpoint, PO may have an incentive to act inappropriately. In these circumstances, PO lacks such abilities and knowledge and might show willingness to agree or, at least, not to resist SO's privileges to make an arrangement upon the size of welfare-pie and to decide its redistribution and delivery. This clarifies players' asymmetric power dynamics.

4.2 Bargaining procedure. We stressed before that in the Welfare State there would always be disagreements about slicing the pie of income taxes. Recall that we consider two fictitious organizations—SO, acting in the role of negotiators over primary goods, and PO, in charge of other public goods. As was the case in the simple game, a precondition for the agreement is that the expectations of negotiators solely depend on efficient policies of SO within the framework of how to set the NIT-gross level parameter, rather than size. As a consequence of this dependence, efficient gross levels provide an efficient correspondence to the slices. Accepting the precondition, PO will only propose efficient slices for the obvious reason that all inefficient NIT-gross levels corresponding to inefficient slices will be certainly rejected. A situation, however, is realistic when PO—by negligence, mistake or some other reason—suggests an inefficient recommendation. We exclude, then, that in response, knowing its own prescription of efficient gross level, SO mistakenly disregards own prescription and accepts the inefficient level anyway. On the contrary, realistic as well, let SO have an intention to disregard an efficient recommendation. Even so, this will not be the case, since in any agreement both players are committed by propositions to slices, no matter what the slices are. Therefore, we believe that PO's recommendation on gross levels makes a rational argument that SO must accept or reject, making a new proposal in the standard way. Such an account explains that the outcome of the bargaining game for both parties might be, instead of an agreement upon slices, a desirable gross level \( \xi^* \in [\xi_1, \xi_2] \). Hereby, only the interval, named also the scope \( [\xi_1, \xi_2] \) of negotiations, bids proposals, which now, by default, are binding efficient slices \( x \), and then the bargaining game performs exclusively in the interval \( [\xi_1, \xi_2] \). Hence, \( [\xi_1, \xi_2] \) is the scope of most trusted welfare platforms for negotiations, where players would choose, accept or reject proposals of efficient NIT-gross levels. Expectations of negotiators depending on \( [\xi_1, \xi_2] \) arrange a bargaining frontier, Fig. 4-5, as a way to assemble the bargain portfolio. Therefore, given the portfolio has changed its color; negotiators may now conceive themselves as making NIT proposals. If rejected, the roles of actors change and a new proposal is submitted. The game continues in a traditional way of alternating-offers.
**4.3 Analysis.** We now proceed to a more accurate analysis of the situation. In accord with bargaining procedure, social organizations propose a slice $x$ of the welfare-pie they commit to, and prescribe an efficient NIT-gross level $\xi$, which is fiscally reasonable to materialize or implement. Public organizations will accept or reject the proposal. In the latter case, they will make an alternative proposal $y$ of a slice they commit to but only recommend $\xi$ as a desirable gross level. Social organizations, in general, are not committed to accept the recommendation $\xi$. Within the scope of negotiations $[\xi_1, \xi_2]$, the recommendation is equivalent to a proposal of the slice $y$ of the pie, since by rejecting the proposal, social organizations would violate the commitment $x = 1 - y$ to the slice they committed to by the agreement. The game continues, whereby each actor takes turns making a proposal, $x$ or $y$ about the slice, and a prescription or recommendation of the NIT-gross level $\xi$ in opposition of the other. When a rejection occurs, the momentary phase of the game consolidates a draft.

Although the rules can be perceived as fiscally safe, the game itself contains a new challenge. Elevated gross level $\xi$ does not necessarily increase the NIT-netto level $u$, as the welfare hazard ($h$-effect) is still present. An increased number of claims may have a declining effect on $u$, whereby low-income agents—the benefit claimants—may claim subsidies. As a result, in contrast to increasing gross levels $\xi$, and despite the required unavoidable increase in taxes, this will decrease the netto level $u$. With the proviso that social organizations commit to slice $x$ and public organizations to $y = 1 - x$, the fiscally safe space of NIT-netto levels $u(\xi, x)$ corresponds to points, which arrange a single $\cap$-peaked curve depending on NIT-gross level $\xi$ as a parameter.

The peak $\xi^o = \arg \max_x u(\xi, x^o)$ of single $\cap$-peaked curve (assuming, as we do, that the maximum can be reached) represents an efficient welfare policy $\xi^o$ of social organizations. Therefore, while the bargain portfolio of social organizations contains efficient policy $\xi^o$ as a function of $x^o$, the portfolio contains also the slice $x = x^o$. Considering the reverse case for any efficient policy $\xi^o \in [\xi_1, \xi_2]$, which corresponds to top value given by $u = u^o$, a unique slice $x^o$ solves $u(\xi^o, x) = u^o$ for $x$; $g(\xi^o, x^o) = g^o$ represents the non-conforming expectation of public organizations. We can thus refer to the slice $x^o$ as an efficient slice representing policy $\xi^o$. Depicted in Fig. 4 in various projections (on expectations $\{u^o, g^o\}$, and at Fig. 5 on wealth amount $W$ and taxes $\tau$), efficient peaks $\xi^o$, which correspond to efficient slices $x^o$, arrange the geometry of what we refer to as the bargaining frontier. This geometry highlights top values $u^o$—efficient policies of social organizations at peaks $\xi^o$.

One, would perhaps recognize here the well-known result (Canto et al., 1981: 11) of the Laffer curve: "The marginal tax revenue raised decreases with increase in tax rates, finally reaching some point where the marginal tax revenue raised is zero. Beyond this point, any tax rate increases will reduce revenue collection." Our result of single peaked expectations of social organizations is similar. First, "NIT-netto level $u$ being proposed in the normal range of NIT parameter $\xi$." Next, "by passing through the top point of $u$ as a function, the proposals $u$ will be assessed and reviewed in the range of prohibited values of $\xi$.

---

10 We already emphasized the worsening quality of the welfare-pie when the NIT-level is "climbing" high.
We have already composed above a fiscally safe average \( B(\xi) \) of subsidies and the average \( W(\xi) \) of taxable income. Now, using this fiscally safe composition \([B(\xi),W(\xi)]\), which contributes to the rules and regulations of the welfare game, the players’ expectations can be set. The composition \([B(\xi),W(\xi)]\), at the end of the subsection, will lead to an appropriately settled bargaining problem that will associate the threat originating from the third partaker—the threat emanating from political and electoral maneuvering of the Voter—with the risk of negotiations to collapse prematurely. We have already dealt with the rules of the game but now we make two rigorous suppositions. Let us first specify the expectations:

**Expectations of Negotiators 1 and 2, and the Voter’s risk factor:**

Negotiator No.1, \( u \) — the NIT-netto level of a fictitious agent (after tax rank), cost of living, or an allocation to fulfil basic necessities of the needy;

Negotiator No.2, \( g \) — all except expenses on subsidies, an account for refund of public goods expenses, expectation that benefits all in the society;

Voter, \( q \) — the risk of premature collapse emanating from the threat of political and electoral maneuvering of citizens facing higher taxes.

Next, we assume that the rules and regulations applicable to slicing the welfare-pie include the volatility constraint \((\ref{eq:1})\), which certifies the fiscally safe composition \([B(\xi),W(\xi)]\) for policy \( \xi \). This assumption depends on the conclusions of the analysis undertaken above, as funding a fiscally safe composition in the welfare game could not be implemented without the volatility constraint \( L(\xi,x,u) = 0 \) (Observation 1).

Furthermore, varying \( \xi \) under their own rules and regulations, let social organizations propose a fiscally safe policy \( \xi = \xi^o \), which, for each slice \( x = x^o \) they commit to, binds \( x^o \) to \( \xi^o \), ensuring efficient allocation of NIT-netto level \( u(\xi^o,x^o) = \max_{\xi} u(\xi,x^o) \); no matter who originates the proposals \( x^o \) or \( y^o \). Clearly, for each slice \( y^o \) proposed by public organizations, social organizations will reject ineffective recommendation if \( \xi \neq \xi^o \); thus, effective policy \( \xi = \xi^o \) must occur amid efficient slices \( x^o \) at \( \{u^o = u(\xi^o,x^o), g^o = g(\xi^o,x^o)\} \) as an ongoing precondition for the agreement; the procedure was already discussed. Thus, social organizations have no reason to reject efficient recommendation \( \xi^o \); otherwise, when \( \xi \neq \xi^o \), they, eventually, cannot keep to efficient commitment \( x^o \).

**Observation 2.** Policies \( \xi \) at the curve \( S_b = \{u(\xi,x),g(\xi,x)\} \) certifying fiscally safe and effective compositions \([B(\xi),W(\xi)]\) of social organizations must satisfy the constraint

\[
D(\xi,x,u) := \frac{\partial}{\partial \xi} L(\xi,x,u) = \frac{\partial}{\partial \xi} \left[(\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi)\right] = 0, \quad (5)
\]

where the curve \( S_b \) is a bargaining frontier. The proof of necessary and sufficient conditions for constraint \((5)\) is available on request.
It is evident that NIT-netto level \( u \) and amount \( g \) of public goods at the frontier \( S_b \) depend exclusively on policies \( \xi \), \( \langle u(\xi), g(\xi) \rangle \) \( \in S_b \). Recall that parties are conceived, then, as themselves making proposals over policies \( \xi \), instead of slice-proposals \( (x,y) \).

Frontier \( S_b = u(g) \) in Fig. 4 illustrates the expectations. Particularly, when

\[
\begin{align*}
Q(\xi, \tau, g) &= 0 \quad \text{Delivery (1)}; \\
L(\xi, x, u) &= 0 \quad \text{Volatility (4)}; \\
D(\xi, x, u) &= 0 \quad \text{Frontier (5)};
\end{align*}
\]

as rules and regulations for the welfare game between political organizations of the Welfare State,

following a succession of collections of sub-expressions and simplifications, and also with the proviso of proportional (flat) taxes, together with the preliminary settings \( \tau'_\xi(x,\xi) > 0 \), \( \tau''_\xi(x,\xi) > 0 \), \( u'_\xi < 0 \), \( u'_\xi(\xi_1, x) > 0 \), \( u''_\xi(\xi_2, x) < 0 \), \( u''_\xi > 0 \), \( u'_\xi > 0 \), \( g'_\xi > 0 \), \( g''_\xi > 0 \), \( g''_\xi > 0 \) or \( g''_\xi < 0 \), see above, the constraints lead to analytical solution:

\[
\begin{align*}
    u(\xi) &= \xi - \tau(\xi) \cdot (\xi - \phi), \quad \tau(\xi) = \frac{z(\xi)}{W(\xi)}, \quad z(\xi) = B(\xi) + g(\xi); \\
    g(\xi) &= \frac{W(\xi)}{v(\xi)} - B(\xi), \quad \text{where } v(\xi) = 1 + (\xi - \phi) \cdot \left( \frac{B'(\xi)}{B(\xi)} - \frac{W'(\xi)}{W(\xi)} \right)
\end{align*}
\]

which turns to be without constraints; mathematical derivation available on request.

Now, the bargain portfolio contains a scope \( [\xi_1, \xi_2] \) for negotiations that parties will follow in the traditional way. Social organizations propose \( \xi \in [\xi_1, \xi_2] \), which public organizations either accept or reject. If rejected, the roles of actors change and public organizations make an effective recommendation—a proposal \( \xi \) to social organizations. The game continues changing roles until a proposal \( \xi \) is accepted. The policy \( \xi \) on poverty, or the NIT-gross level, is a control parameter of policy rules and regulations. Hereafter, the efficient policy \( \xi \) is referred to as a proposal of policy on poverty when the bargaining over slices \( (x,y) \) of the welfare-pie in eventual agreement is under negotiations.

Before we go any further, let us recollect the threat phenomenon created by the Voter elevating the risk of premature collapse of negotiations. Recall that rejecting proposals, negotiators consolidate a draft, knowing that it is risky to continue the bargain. The Voter may emanate a threat to vote against the draft putting the negotiations at risk of collapse when negotiators try to continue bargaining over too costly proposals without fulfilling the terms of the Voter. The game ends with disagreement or breakdown. Similar to Osborn and Rubinstein (1990: 72), "We can interpret a breakdown as the result of the intervention of a third party, who exploits the mutual gains. A breakdown can be interpreted also as the event that a threat made by one of the parties to halt the negotiations is actually realized. This possibility is especially relevant when a bargainer is a team (e.g. government), the leaders of which may find themselves unavoidably trapped by their own threats."

\[\text{11} \quad \pm \text{ rates } W'(\xi) \leq 0, \; W''(\xi) \geq 0 \text{ of changes in wealth amounts } W(\xi) \text{ are essential for the analysis, whereas function } B(\xi) \text{ is valid only with } B'(\xi) > 0, \; 0 < \phi < \xi.\]
Suppose that negotiators bargain over all fiscally safe policies $\xi \in [\xi_1, \xi_2]$ within the scope of negotiations $[\xi_1, \xi_2]$. Below, we follow the alternating-offers game $\Gamma(q)$ with an exogenous risk $q > 0$ of premature collapse (Osborne and Rubinstein 1990: 71-76). Each time the proposal $\xi$ is rejected by one of the negotiators, the momentary phase of the game consolidates a draft. As mentioned before, the Voter emanating a threat may vote against the draft. In these circumstances, the negotiators might feel uncertain of how to proceed if the terms of voter-citizens (taxpayers) are not met: negotiators might walk away from the bargaining table prematurely. Extracted from the endpoints $\xi_1 < \xi_2$ of bargaining frontier $S_b$, the worst-case scenario $\{(u_1 = u(\xi_1), g_1 = g(\xi_1)), (u_2 = u(\xi_2), g_2 = g(\xi_2))\}$ naturalizes this risk $q$.

A closer look at what is known (since Roth 1977) as the "well-defined bargaining problem", or individual rationality, associated with the Nash bargaining scheme $(S, d)$, seems to be instructive. Indeed, inequalities $g_1 > g_2$, $u_1 < u_2$ hold for the pair $d = \langle d_1 = u_1, d_2 = g_2 \rangle$ merging the bad ends $\langle u_1, g_1 \rangle$ and $\langle u_2, g_2 \rangle$ of both parties at the scope $[\xi_1, \xi_2]$. The point $d$, introduced here, naturalizes the Nash disagreement point of the problem $\langle S_b, d \rangle$, $S_b \subset \mathbb{R}^2$. Thus, compared to traditional approach of compact convex set $S \subset \mathbb{R}^2$, inequalities $s > d$ are also true for any pair $s \in S_b$. Therefore, the pair $\langle S_b, d \rangle$ for the bargaining frontier $S_b$ becomes a well-defined bargaining problem. But, it is not immediately apparent whether the point $d$ is a fiscally safe outcome. The following observation rules out such a doubt.

**Observation 3.** To testify whether the point $d = \langle d_1, d_2 \rangle = \langle u_1, g_2 \rangle$ becomes a fiscally safe outcome of the welfare-pie game, it is necessary and sufficient that there exists a policy $\delta$ on poverty, which resolves the equation:

$$\left(\delta - \phi\right) \cdot \left(B(\delta) + d_2\right) - \left(\delta - d_1\right) \cdot W(\delta) = 0;$$

condition $\delta \notin [\xi_1, \xi_2]$ is necessarily required.

Notice that in the worst-case $\delta$, the average wealth amount redistributed in society is $W(\delta)$ and the average of expenses for funding subsidies is $B(\delta)$. Proposal $\delta$ depends on endpoints of the bargaining interval $[\xi_1, \xi_2]$. This dependence, provided that Equation (6) can be solved for $\delta$, serves as the basis for validation of pre-equity condition of breakdown, covered in Section 6 below.

Finally, what follows is the proper choice of expectations in the alternating-offers game with a risk $q$ of premature collapse of negotiations. The choice solution of the game encapsulates, as given by Osborn and Rubinstein, (1990: 75), the bargaining power $\alpha$ of negotiators for an appropriately settled bargaining problem.

**Observation 4.** In the alternating-offers game $\Gamma(q)$ with the risk $q > 0$ of negotiations to collapse prematurely into breakdown point $\langle d_1, d_2 \rangle$, the functions $(u(\xi) - d_1)^{\alpha}$ and $(g(\xi) - d_2)^{\alpha}$ imply the expectations of social and public organizations respectively. The solution $\lambda$ of the well-defined bargaining problem $\langle S_b, d \rangle$ is close to the pair $(\lambda_1, \lambda_2)$, $\lambda_1 \leq \lambda \leq \lambda_2$, thus, solving equations $(1 - q) \cdot (u(\lambda_2) - d_1)^{\alpha} = (u(\lambda_1) - d_1)^{\alpha}$ and $(1 - q) \cdot (g(\lambda_2) - d_2)^{\alpha} = (g(\lambda_1) - d_2)^{\alpha}$ for variables $\lambda_1, \lambda_2$; without proof.
5. Political and electoral maneuvering

Only the voting results can reveal the true incentives of citizens who give the democracy its final judgment. Only through the democratic process can the voters step up in the roles of actors to whom the opportunity is granted to govern organizations that intend to act in people's interest through social planning to allocate and redesign public resources. It is voters' inequalities, life plans, social class and origin, native endowments, political capital, etc., that determine what bulletin to collect at the voting table. As a consequence, voters' reasonable disagreements or interpretations of reality that affect the voters' own choice would have an impact on political and electoral maneuvering and, thus, the voting result. Given unpredictable deviations in voters' political views and opinions of how the income redistribution ought to be organized, the results are not fully predictable. The problem stems from the fact that welfare proposals that benefit all sometimes require higher taxes. Political organizations would in no way be confronted directly with taxpayers' selfish attitudes toward lower taxes. Such an attitude deserves, perhaps, critical examination of both taxpayers and representative voters' roles. Given these points, our question is: Why should political organizations in the roles of welfare game actors as rational negotiators care about lower taxes?

5.1 Unanimous consent. The situation is well suited to introduce political and electoral maneuvering of taxpayers with an external risk \( q, 0 < q < 1 \), of premature collapse of negotiations. Indeed, Fig. 5 depicts the curve of efficient welfare policies/proposals \( \xi \) upon NIT-gross levels in the bargaining game \( \Gamma(q) \). Fiscally safe to finance the expectations of our rational negotiators, proposals \( \xi \), forming the curve, have been projected onto the two-dimensional space of taxable income (the wealth amount) \( W \) and the tax rate \( \tau \). Although the pair \( \langle u, g \rangle \) is embedded in each point, it is not visible on the graph. These invisible pairs in the upper part of the graph symbolize lower basic (primary) yet higher public (non-primary) goods, whereas the pairs in the lower part symbolize lower public but higher basic goods. Thus, in voters' view, once all views are represented, the expectations \( \langle u, g \rangle \) of negotiators for fixed tax \( \tau \) are more favorable for some coalitions of voters, compared to other coalitions. Hereby, efficient NIT-gross levels resulting from eventual agreements between negotiators are two-fold. Some voters will accept higher, whilst others support lower taxes. Unless the tax is excessively high, the "upper coalitions" of voters will always try to outmaneuver the "lower coalitions." The negotiators are aware about these outmaneuvers when taxes are high. To be trapped in negotiations, as they believe might happen in the bad end, the result might be withdrawal or defection placing the negotiations at risk \( q > 0 \) of premature collapse. The negotiators expect the range of eventual outmaneuvering to shrink or even to vanish on the lowest tax. Therefore a desirable outcome for all might be a lower tax.

N.B. The lowest level is just one of various fiscal policies that have an effect on motions to pass by unanimous consent.
An issue of avoiding, if possible, the risk \( q > 0 \) of premature collapse has not yet been addressed. Assume that a number of efficient welfare policies are waiting to be put into the bargain portfolio. Clearly, in voters' view, the minimum tax is the most desirable policy. The minimum tax, in spite the negotiators know voters' attitude toward higher taxes, might not necessarily be desirable outcome of negotiations. Therefore, the negotiators can go against voters' preferences because the bargaining power of social and public organizations, as negotiators, might be strong enough for negotiating selfish decisions favorable either for both sides alone. Thus, to avoid outcomes resulting in collapse, the indicator \( \alpha, \, 0 < \alpha < 1 \), of social organizations bargaining power, where \( 1 - \alpha \) signifies the bargaining power of public organizations, comes into consideration. As was the case in our simple game, toward a desirable half of the pie, we wish to adjust the indicator \( \alpha \) in a way that it will help the social planning by redesigning and rebalancing (normalizing) the bargaining procedure between social and public organizations. The rules and regulations of social and public organizations activities must be redesigned accordingly. Hereby, akin to the simple game described above, two highly pragmatic negotiators will try to slice the welfare pie in the same manner.

Let the social and public organizations try to slice the pie in more a precise voting scheme. Let the Voter, who represents taxpayers refunding the pie, votes only for or against the draft, accepting or rejecting the phase of the negotiations archived at the moment. The policy \( \lambda \), which minimizes the tax burden, is what voters are seeking. In accord with the above, we can redesign the bargaining power \( \alpha \) of social organizations to ensure that the slicing of the welfare-pie would incorporate the Nash axiomatic solution \( \lambda \) into bargain portfolio as an outcome. In doing so, (Osborn and Rubinstein 1990: 75), the risk of premature collapse of negotiations vanishes while \( q \to 0 \) at the outcome \( \lambda \). That is the case as it appears.

In this case, a tax policy in which no one objects to a proposal is desirable. To be implemented by all members of the society, the marginal wealth-tax minimum at the bargaining frontier might be the voters' desirable preference. Extracting the expression \( \tau(\xi) \) from (1), in accordance with our analytical solution without constraints, the frontier \( S_b = u(g) \) is a curve \( \{u(\xi), g(\xi)\} \) that embodies voters' preference \( \tau_{\text{min}} \), which becomes:

\[
\min_{\xi \in [\xi_l, \xi_u]} \tau(\xi) \equiv \frac{B(\xi) + g(\xi)}{W(\xi)}.
\]

With the proviso that \( \tau(\xi) \) is smooth enough, the solution of the voter problem is the root \( \lambda \) of the equation \( \tau'(\xi) = 0 \). Root \( \lambda \) allows the social planning of the rules and regulations according to \( \alpha \) in a way that the negotiation power \( \alpha \) of social organizations during negotiations is sufficient to persuade public organizations to agree upon NIT-netto level \( u(\lambda) \).
In this way, the old standard in our simple game, of how to socially plan the bargaining procedure, can be now a new realistic standard in our welfare game. Indeed, let \( \alpha \) facilitate negotiating power of SO, and \( f(\xi, \alpha) = (u(\xi) - d_1)^\theta \cdot (g(\xi) - d_2)^{\varphi} \). Recall, that we first took the derivative of \( f(\xi, \alpha) \) with respect to \( \xi \) evaluating \( f'_\xi(\xi, \alpha) \); then we replaced \( \xi = \frac{1}{2} \). Instead, replacing \( \xi = \lambda \), we must now be more realistic social planners. This more realistic power \( \alpha \) of social organizations must solve the equation \( f''(\xi_{\text{eq}}, \alpha) = 0 \) for \( \alpha \).

**Observation 5.** Provided negotiators can reach a preliminary agreement on wealth-tax \( \tau = \tau(\xi) \), condition \( \lambda = \arg \min_{\xi \in [\xi_l, \xi_u]} \tau(\xi) \) is necessary to put forward a poverty proposal \( \lambda \) before voter-citizens by unanimous consent. At the bargaining frontier \( S_b \), proposal \( \lambda \) outlines a unique outcome \( \phi, \xi \Rightarrow z, x, \tau(\lambda), \{u(\lambda), g(\lambda)\} \in S_b \).

### 6. Pre-Equity condition of breakdown

Traditionally, in the alternating-offers game, the breakdown point—when one of the players’ decides to break or defect—corresponds to two pairs of payoffs \( \{10, 0, 1\} \) (Osborn and Rubinstein 1990: 73). In the welfare game, due to external press of voters, when one player cooperates, there always will be a temptation, for the other to defect, 13 placing the negotiations at risk \( q > 0 \) to collapse prematurely. Therefore, when the premature collapse occurs, it is important to arrange the terms of contract in such a way that players cannot exploit or misuse the collapse environment to their own advantage. The quality and the size of the welfare pie in the event of collapse should be equal for both players because one of the players will possess the entire pie independently of the environment. Whatever the collapse environment seems to embrace, we are working here toward endogenous form for equity in accordance with players’ non-conforming expectations normalizing the breakdown point under the description valid for the alternating-offers game \( \Gamma(q) \).

As previously stated, the standard breakdown point in the alternating-offers game corresponds to a pair of pairs \( \{10, 0, 1\} \) of utilities. In this form, the point is generally found using ex-ante linear transformation—exogenous normalization of utilities. The standard point exposes equity of players when the collapse of negotiations is looming closer upon which the players cannot make binding agreements. When the most unfavorable result of negotiations is a reality, the collapse includes additional parameters—the tax \( \tau \) and taxable income \( W \). In order to equalize (endogenously normalize) the point, the players can commit to or agree upon the normalization of additional parameters a priori. Without existence or warranty of such a pre-equity, an endogenous normalization is unrealistic. Therefore, in view of taxpayers’ political and electoral maneuvering, even if the pre-equity normalization is not always possible, it is certainly more constructive to set the breakdown point according to some rational principle.

13 Like to defect in prisoners dilemma
Before going into details of the principle let us recollect the collapse environment once again. Recall that at the start of the negotiations, the draft of a contract covers taxes \( \tau \) and the wealth amount \( W \). The product \( \tau(\xi) \cdot W(\xi) \) identifies the size \( z \) of the pie within an interval \([\xi_1, \xi_2]\) —the scope of negotiations. The scope \([\xi_1, \xi_2]\) establishes the boundary for negotiators, i.e., the initial, most positive proposal \( \xi_1 \) for public organizations (and the most negative for social organizations), whereas proposal \( \xi_2 \) yields exactly the opposite outcome.

In other words, \( u_1 = u(\xi_1) \) is the most favorable outcome for one side, while \( g_1 = g(\xi_1) \) is the most unfavorable for the other, whereby \( u_2 = u(\xi_2) \) and \( g_2 = g(\xi_2) \) can be paired in the same but inverse way. We will show that the breakdown point \( \{(u_1, g_1), (u_2, g_2)\} \) may be conditionally, albeit endogenously, encoded into the income distribution samples \( P(\sigma, \xi) \).

As promised, we now contribute to implementing a rational principle of how to set the breakdown endogenously. We consider a situation driving the welfare policy in the context of cost-benefit analysis. In case the collapse of negotiations is looming closer, the differences in the amounts of wealth and taxes for funding low-cost welfare policy \( \xi_1 \) against an expensive policy \( \xi_2 \), \( \xi_1 < \xi_2 \), i.e., funding expectations \( (u_1, g_1) \) for \( \xi_1 \) against \( (u_2, g_2) \) for \( \xi_2 \), \( u_1 < u_2 \), \( g_1 > g_2 \), can amplify misunderstandings and contribute to traps. For example, at the endpoints of the scope of negotiations \([\xi_1, \xi_2]\), the public spending \( z(\xi_1) \), \( z(\xi_2) \), or the sizes of the welfare-pie, can require delivery of the same wealth amounts \( W(\xi_1) \) and \( W(\xi_2) \) but for different prices as taxes \( \tau(\xi_1) \) and \( \tau(\xi_2) \). Therefore, prior to the start of the game, the expectations \( s_1 = (u_1, g_1) \) and \( s_2 = (u_2, g_2) \) in the most unfavorable circumstances should preserve equal prices \( \tau \) for the delivery of equal amounts \( W \) of wealth. Such a market driven interpretation, like delivery of commodities to their final destination, relies heavily on the definition of the size of the pie, which equals to \( \tau \cdot W \). The interpretation is only applicable to the case of flat (proportional) taxes, cf., Buchanan (1967: 4.7.1).

To explicate the interpretation of reasoning in previous lines, let us turn to the Fig. 5 once again. Based on the above, our goal is to fix an interval \([\xi_1, \xi_2]\) solving a system of two non-linear equations, \( W(\xi_1) = W(\xi_2) \) and \( \tau(\xi_1) = \tau(\xi_2) \) by trying to find a cross point \((W^*, \tau^*)\) where the bargaining frontier crosses its own contour on the plain with \( W, \tau \) as \( XY\)-axis coordinates, i.e., the roots \( \xi^*_1 \) and \( \xi^*_2 \). Although the calculus of the point \((W^*, \tau^*)\) is realistic, it does not confirm the possibility of normalization in general. We do not claim that the equity condition can be reached in all circumstances. In a number of examples where the validity of the condition was detected, we found a breakdown endogenously encoded into income distribution \( P(\sigma, \xi) \) —the normalization in the form of \( \{(u_1^*, g_1^* = g(\xi_1^*)), (u_2^*, g_2^* = g(\xi_2^*))\} \).
**Summary.** In order to bring the negotiators, if possible, into just and equal positions prior to negotiations, a rational starting point might be in equalizing wealth amounts $W$ and taxes $\tau$ in the collapse environments $\xi_1$ and $\xi_2$. Endogenously encoded into income distribution $P(\sigma, \xi)$, we label the equity condition $[\tau(\xi_1) = \tau(\xi_2), W(\xi_1) = W(\xi_2)]$ as a pre-equity of breakdown. Particularly, if valid, the pre-equity will equalize fiscally realistic and just demands for public spending prior to negotiations—the size of welfare-pie: $z(\xi_1) = z(\xi_2)$.

7. Discussion

One possible way to show the true essence of the economic reality behind the welfare game could be answering the following question: *Is it true that funding subsidies and keeping the budget fiscally safe is going to be difficult to sustain when the tax burden for all citizens is decreasing?* On the surface, it seems that, at some point, fairness and equity may disappear because the "rich simply get richer and the poor get poorer." The effect of "tax relief for the rich" seems to affect the welfare adversely. In the face of these controversies, no one can estimate the extent of potential fallout that may result from such outcomes. We will leave the answer to the reader as a voter who can decide what needs to be done in order to socially plan and redesign the welfare system and public services. Taking advantage of this opportunity, it is instructive to perform an exercise related to the most appropriate choice of welfare policy, as shown in a column "minimizing wealth tax" of Table 1, created through numerical simulation carried out upon fictitious income redistribution of agents. We estimated that tax relief for all, minimizing wealth tax, in fact, is fiscally safe but at the same time, ensures just and fair income redistribution.

The discussion below deserves, perhaps, some emphasis on assumptions made during the analysis. Our assumptions have been purely normative, i.e., "what ought to be" in economic or political matters, as opposed to "what is". Therefore, despite the fact that it was not clear what the actual case was, our theoretical results rest on the following assumptions.

First, we supposed that negotiators playing the game could come to an agreement. Possible agreement could lead to a just solution in accord with rules and regulations of the game, particularly, to just and fair poverty rate. However, although it is profoundly mistaken idea of justice, occasionally, it is taken willingly. In Table 1, we present the percentage of agents below the NIT-gross level establishing the poverty rate. A willingly driven initiative could reduce the rate below 0.41%, which wrongly appears to be the most just and fair!

---

14 Recall that the NIT-gross level determines who is who—who is rich, i.e. has income higher than gross level, and who is less fortunate with an income less than the level by definition.
Second, the welfare-pie redistribution compensated for the inequalities of agents' incomes below the NIT-gross level that, together with similar parameters, has been set by the national government. Taking into account increases in the cost of living, the official number of individuals living in poverty supposed to be adjusted annually according to poverty guidelines. Although our key assumption was that the rules of public organizations in the welfare game inherited no more than an advisory authority, the rules and regulations that govern decision on the NIT-gross level have been moved under the mandate of social organizations. In fact, in the analysis, we were trying to emphasize the distinctions of stereotypes about the motivations of social and public organizations. In our view, social protection that is most likely to be just as fair should be separated as an independent institution, or better yet, as an assembly of independent organizations or programs. We believe that only in case of organizational independence social protection could be expected to yield effective social policies. It is only when these guidelines of independence are applied that the value judgment below makes sense. Therefore, in reaching efficient policy on poverty, we concluded that social organizations should be in a privileged position, which allowed them to prescribe the NIT-gross level independently. Still, it should be noted that a characterization of whether setting up such a privilege was a positive or negative restriction requires additional investigation.

Next comes the power indicator $\alpha$, which highlights the amount of resources, skills and competence of social organizations, etc., to justify and maintain social duties under the principle of how the State ought to act when trying to fulfill its welfare mission. The fundamental factor in our analysis was the social protection of the society, as a whole. When the decision made is not in line with the wishes of special interest groups, as pointed earlier, social protection could be a never-ending theme in political debates. A conflict of interests might lead to violent upsets, raising an opportunity for making policy in favor of these groups. According to the foregoing account, and previous observations pertaining to the independence of the social system as a whole, we believed that there was no reason for the citizens to have advanced social organizations, which require considerable administrative efforts and fiscally unrealistic expenses. Thus, with the reference to the vital role of public organizations in the society—due to their central position of purchasing and delivering public services—, we have seen in the interpretation of the parameter a value to impose a lower grade $\alpha$ to social organizations and a higher grade $1 - \alpha$, $0 \leq \alpha \leq 1$, to public organizations. It was, therefore, reasonable to assume that social organizations, with almost no extra effort, would be in "ample degree armed" to make efficient decisions. Herewith, in planning and adjusting the size of the pie, aiming to settle and assist the State social mission to suit a fiscally realistic welfare policy, we tried to redesign the balance of power between social and public organizations by adjusting the power indicator $\alpha$. In doing so, in order to benefit all in the society, we could, in our view, adjust the rules and regulations of the State closer to the legal responsibilities and moral obligations of its citizens.
The last assumption was the unanimous consent—a situation in which no one can find a reason to object the motion to the agreement. Reaching a unanimous consent is a difficult and time-consuming process, as prolonged negotiations may not be in the interest of anyone, even if the decision is finally reached. Therefore, we supposed that political and electoral maneuvering of voters against higher taxes might put the negotiations at risk of premature collapse. Particularly, an acceptable point of view was that there existed an undesirable risk of negotiators to defect—one that could interrupt negotiations ahead of schedule. Hereby, we brought the problem of negotiation collapse environment into focus. In case that the failure of negotiations for both parties is extremely undesirable, we hoped that the negotiators would move faster toward, or be close enough to a common denominator called the consensus, where all are approaching each other making concessions. In view of social clients receiving subsidies, a policy of higher tax rates might be the most favorable and just solution. Yet, in view of the consumer, the minimum tax rate was out of the question. Therefore, for those agents who finance subsidies, as we assumed in the analysis, the minimum tax rate was a just and fair redistribution of wealth, not least that it was, also, an outcome in which no one objected to a proposal visualizing the society common denominator. Assuming, as we already did above, that agreement upon the rules and regulations of the game could be reached, the outcome, which minimizes taxes, offered a vision of what policy should entail. Such an outcome was, in general, as we supposed, worth the time and effort even if the vision was a utopia.

Table 1, by presenting all these four assumptions, suggests proposals—columns laid before voter-citizens to vote on. Note, that voting for the policy $\eta = 79.23$ with equal power of negotiators is less just and less fair than the policy $\lambda = 45.50$, where the minimum 26.52% of taxes is reached. Thus, only the policy $\lambda$ on NIT-gross level (Fig. 5) has a chance to pass by unanimous consent. Indeed, in the variety of welfare game regulations, when engaged in an interaction to implement equal policy $\eta$ (as was the case when PO and SO engaged to obtain a slice of welfare-pie in the simple game), the equal power $\alpha = 0.5$ of social organizations’ was stronger than 0.21. However, the incident with weakened power $\alpha = 0.21$ is yet to be determined and the aim of the customers can still be reached by applying the policy $\lambda = 45.50$ for the tax rate $26.52% < 28.21%$. Thus, by reduction of taxpayers‘ obligations—even with SO’s weakened bargaining position—SO will be able to come to a desirable agreement with public organizations to maintain the most just and fair NIT-gross level of wealth for all.

Summary. In closing the discussion, we would like to point to a decision that embodies the breakdown of negotiations. Utopian society, planned according to breakdown, as shown in Table 1, seemingly ignores the social protection because almost all citizens are considered rich by default; poverty does not exist. Given this utopian society financing expenses on vital public/non-primary goods, the breakdown solution on equity condition requires $-2.49$ public debt per capita. This will, in response, require borrowing or creating money refunding the debt. We, therefore, admit that a self-financing tax system has a better chance of being implemented based on the lowest tax burden 26.52%.
8. Concluding remarks

It is time to review the knowledge that the welfare game brought to us. We followed the welfare policy treating and reimbursing the less fortunate fairly. By pushing along the edge of NIT-gross level—from the viewpoint of the cost of living—we decided to whom primary or basic goods should be redistributed. Expenses on subsidies for basic (primary) goods and expenses on public (non-primary) goods were estimated separately, and transformed later into functions of the gross level as a parameter of the game. Elevated gross level gave rise to inverse working incentives, referred to as welfare hazard (feedback effect) or $h$-factor, which unbalanced the account for the delivery of all public goods to the end destination. For this reason, a balanced account for the delivery of public goods becomes crucial in resolving political conflict/game between social and public organizations.

Setting up the game, incomes represented the self-interest of agents. Next, we addressed the public finance problem of income redistribution among benefit claimants with lower incomes against those with higher incomes. The goal was to find a solution in the alternating-offers game with two actors having conflicting interests and representing citizens pursuing their own causes. Social organizations argued to put in all efforts to increase the NIT-gross level, whereas the public organizations' expectation—in response to the public wants—was to meet the need for non-primary goods. The threat of political and electoral maneuvering against higher taxes places the negotiations at risk of premature collapse. Thus, the risk of collapse, in favor of taxpayers, was the only driving force in reaching the agreement. In doing so, we gave credit to the tax system to guarantee reasonably high NIT-netto level. This led to the conclusion that, in contrast, arguments demanding an increase in the gross level were weak, since too costly proposals of subsidizing an increased number of low-income claimants for benefits require a raise of financial support. An increased number of claims may have a detrimental effect on the quality of social services guaranteed to the needy. Hereby, if the decline is undesirable, an increase of taxes cannot be demanded, and we must restrict the scope of negotiations to a realistic interval of NIT parameter related to our bargaining procedure.

Given the above, a pretext for the analysis of the domain and the extent of bargain portfolio of organizations in the Welfare State was established. The actors representing organizations were supposed to inherit non-conforming expectations. Instead of decisions on slicing the surplus of income taxes, we provided a guide to how the decisions ought to be analyzed and interpreted within the scope of negotiations at the bargaining frontier. The bargaining power of organizations yielded by the solution of the bargaining problem was used as indicator of social planning of organizations in compliance with citizens' desired visions and ambitions.

It was initially deemed that, due to the uncertainty in the selection of the breakdown point, the model could only treat the power indicator as a variable given exogenously. However, a condition—at least true in valuable examples—that can encode the power indicator endogenously, was found and named "the pre-equity of breakdown." Last, but not least, although few examples clearly cannot make a trend, we presented evidence for the claim that recognized NIT-gross level defined as 50% of the median income, is close enough to be considered as realistic gross level, minimizing the wealth-tax.
8.1 Postscript. This paper encapsulated our long-term vision and beliefs of voter-citizens, arguing as three distinct entities, about the level of basic goods, amount of (non-primary) public goods, and taxes. Although no empirical or conceptual support is given for the key elements of the scheme, a fundamental difference between the social security system and the public sector providing vital public services was assumed. The difference from a more primitive utility specification of monetary functions was not sought. For example, the paper did not address the issue of how the income of households is assembled and utilized, and why the redistribution of equivalently valued non-primary but vital public goods and services cannot be a perfect substitute. All these and similar questions are left unanswered. In short, we did not offer a debate on what was right or wrong in the economic or political environment involving two negotiators and the Voter. Nevertheless, setting up a list of rules and regulations of how to coordinate the game, an addendum below might be informative, since these rules might lead to a well-defined bargaining problem. Given that we solved the problem analytically, the solutions, put differently, assign a number of appealing interpretations within the interval of NIT-gross level parameter establishing the main result of the paper.

8.2 Rules and regulations of welfare policy game

1. The administration of social organizations knew the true and exact incomes of social clients, and, thus, it was required to implement an appropriate auditing regulation.

2. The behavioral pattern of agents remained endogenous. Agents demarcated themselves as benefit claimants in compliance with current rules and regulations related to whether or not to compensate with subsidies those less fortunate who are down on luck.

3. The regulation and maintenance of the balance between debts and credits for funding subsidies and benefits was crucial. Debts and credits remained balanced (fiscally safe) throughout, and in spite of volatility in the economy.

4. Subsidies may become progressively attractive to those benefit claimants for whom it does not pay to "work harder and longer." Likely to be the result of the inverse working incentives of claimants, their claims, are shifting the subsidy budget out of balance. In this context, idempotent rules and regulations were necessary to keep the balance, i.e., to neutralize the welfare hazard ($h$-factor) effect. Adoption of idempotent rules on delivery of public goods predicts and enforces a fiscally safe policy of public spending.

5. Rules and regulations of taxation: (a) exclusively proportional (flat) tax, (b) tax schedules equaled taxable income, and (c) the income taxes accumulated via tax schedules was spent entirely on public needs—the delivery of basic and public goods has reached its end.

6. Social organizations' conduct over NIT-gross levels was self-dependent in a sense that negotiators were committed to the agreement of how to slice the welfare-pie. Internal rules and regulations, allowed social organizations to guarantee independently an efficient level of basic goods. Public organizations' conduct over the gross levels was only in an advisory capacity.

7. In any variety of rules and regulations of how to extrapolate and assess income taxes, the income distribution was considered the only legal repository for tax revenue obligations.
References


Narens, L. and R.D. Luce, 1983. How we may have been misled into believing in the Interpersonal Comparability of Utility, Theory and Decisions 15, 247–260.


Appendix.


We proceed with a specific allocation of the Welfare State encapsulating samples of income distribution, parameterized by NIT-gross level $\xi$, similar to an exponential function:

$$ P(\sigma, \theta + h \cdot \xi) = \frac{1}{(\theta + h \cdot \xi) \cdot \Gamma(m)} \left( \frac{\sigma}{\theta + h \cdot \xi} \right)^{m-1} \exp \left( - \frac{\sigma}{\theta + h \cdot \xi} \right) $$

where

additional ex-ante parameters are: $\theta = 61.9$, $m = 2.07$, $h = -0.18$. Parameter $\theta$ controls the wealth of agents—a horizontal shifts of samples; $m$ controls inequality—a vertical shifts; $h$ is welfare hazard parameter; $\Gamma(m)$ is an extension of $(m-1)!$ to real numbers.

The density function $P(\sigma, \theta + h \cdot \xi)$ reflects the initial redistribution of wealth through tax channels depending on $\xi$. Political decision $\xi' > \xi$ shifts the distribution $P(\sigma, \theta + h \cdot \xi)$ horizontally to the left—toward less wealthy allocation $P(\sigma, \theta + h \cdot \xi')$. When shifted, the distribution $P(\sigma, \theta)$ hides welfare hazard $h$-factor, $h = 0$, of benefit claimants. The rate of change $Hz(\xi) = h \cdot \dot{a}(\theta + h \cdot \xi) < 0$ quantifies a fiscally safe hazard $h < 0$ of the policy $\xi$.

The sample $\xi = \frac{1}{2} \mu$ (median income=\mu) can be made into Lorenz Curve, where agents below an income $151.48$, i.e., 75% of the population, have $51.11\%$ of a total cumulative income, while the rest $25\%$, with incomes not less than $151.48$, have $48.89\%$; Gini Coefficient equals $0.37$. Horizontal shifts do not affect the Gini coefficient, while the vertical do. A more detailed example is available on request.
**A2. Simulation.**

In order to perform simulations, the formulas for average $B(\xi)$ of expenses on subsidies and average taxable income—the wealth amount—$W(\xi)$ may be given over income distribution samples $P(\sigma, \theta + h \cdot \xi)$ in a more realistic but general form:

$$B(\xi) = r \int_0^{\xi} (\xi - \sigma) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma; \quad r \cdot (\xi - \sigma)$$

is a subsidy payment, $0 < r < 1$;

$$W(\xi) = \int_0^{\xi} (\sigma + r \cdot (\xi - \sigma) - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma + \int_{\xi}^{\infty} (\sigma - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma.$$

The total income $\sigma + r \cdot (\xi - \sigma)$ of the needy is obligated for taxation in accordance with the conventional practice; $h$-factor reveals the inverse working incentives—the feedback of social clients. This obligation is in line with our tax amendment to Friedman NIT proposals.

It is worth the effort to validate that a disagreement policy $\delta$ under the primacy of equity principle of breakdown might be an outcome of the game. There is no reason why the equation $(\delta - \phi) \cdot (B(\delta) + d_2) - (\delta - d_1) \cdot W(\delta) = 0$, in accordance with Observation 3, should have a solution in general. For the particular example of income distribution samples

$$P(\sigma, \theta + h \cdot \xi)$$

equals $a(\theta + h \cdot \xi) = \int_0^{\infty} \sigma \cdot P(\sigma, \theta + h \cdot \xi) \, d\sigma$.

---

**Figure 3.** At the sample $P(\sigma, \theta + h \cdot \theta/2, \mu)$ of income distribution density, $\mu$ resolves the equation $\int_0^{\xi} P(\sigma, \theta + h \cdot x) \, d\sigma = 0.5$ for $x, \mu = 82.30$.
Given monetary expectations \( \langle u, g \rangle \) at the endpoints \( \langle u_1 = 6.44, g_1 = 47.18 \rangle, \langle u_2 = 89.26, g_2 = -2.49 \rangle \) of the scope of negotiations—within the interval \( [\xi_1 = 8.00, \xi_2 = 144.54] \)—one can discover that the pair \( d = \langle d_1 = u_1, d_2 = g_2 \rangle = \langle 6.44, -2.49 \rangle, \ u_1 < u_2, \ g_1 > g_2 \) consolidates an equity for breakdown policy \( \delta = 6.39 \in [\xi_1, \xi_2] \); the wealth \( W^* = 120.46 \) and the tax \( \tau^* = -2.06\% \).

The "eventualities of the welfare-pie game", i.e., the magnitude and dimension of poverty proposals, ought to be debated or implemented, will be illustrated in the table below.

Recall already known proposals for incomes \( \eta, \lambda_2, \lambda, \delta \) (whereby \( \delta \) is outside of the scope of negotiations \( \delta \not\in [\xi_1, \xi_2] \)); and a poverty proposal \( \frac{1}{2} \mu \), as follows:

- \( \eta \)  the policy on poverty with equal power of negotiators; the social and public organizations are in symmetric positions or equal roles;
- \( \lambda_1 \) an alternating-offer of social organizations—what the public organizations accept;
- \( \lambda \)  the policy on poverty minimizing wealth-tax;
- \( \frac{1}{2} \mu \)  50% of the median income; above income \( \mu \) we have a half, and below that \( \mu \) —another half of agents;
- \( \lambda_2 \) an alternating-offer of public organizations—what the social organizations accept;
- \( \delta \) the most negative outcome: the breakdown policy or disagreement, which naturalizes the risk of negotiators' premature collapse, caused, for example, by mutual traps.

### Table 1. Numerical simulation behind the welfare policy dilemma of a negative income tax system; SO—Social Organizations, PO—Public Organizations

<table>
<thead>
<tr>
<th>Obtained by means of income distribution density (Fig. 3); personal allowance ( \phi = 4.03 ); ( \theta = 61.9; \ h = -0.18; \ m = 2.07; \ r = \frac{3}{4} ); a proportion to ( (\xi - \sigma) )</th>
<th>Policy of equal, symmetric power of negotiators</th>
<th>SO proposal accepted by PO</th>
<th>Proposal minimizing wealth tax</th>
<th>Gross level = to 50% of median income</th>
<th>PO proposal accepted by SO</th>
<th>Policy of disagreement, the breakdown</th>
</tr>
</thead>
<tbody>
<tr>
<td>NIT-gross level; welfare policy ( \xi )</td>
<td>79.23</td>
<td>40.79</td>
<td>45.50</td>
<td>41.15</td>
<td>50.28</td>
<td>6.39</td>
</tr>
<tr>
<td>Poverty rate: percentage of agents below the NIT-gross level</td>
<td>47.36%</td>
<td>15.73%</td>
<td>19.15%</td>
<td>15.99%</td>
<td>22.81%</td>
<td>0.41%</td>
</tr>
<tr>
<td>Negotiating power of social organizations ( \alpha(\xi) )</td>
<td>0.50</td>
<td>0.18</td>
<td>0.21</td>
<td>0.18</td>
<td>0.24</td>
<td>Not defined</td>
</tr>
<tr>
<td>NIT-netto level; the after tax of ( \xi ) ( u(\xi) )</td>
<td>58.02</td>
<td>31.02</td>
<td>34.50</td>
<td>31.29</td>
<td>37.99</td>
<td>6.44</td>
</tr>
<tr>
<td>Account for public goods expenses ( g(\xi) )</td>
<td>19.02</td>
<td>27.63</td>
<td>26.70</td>
<td>27.56</td>
<td>25.75</td>
<td>-2.49</td>
</tr>
<tr>
<td>Account for subsidies transfers ( B(\xi) )</td>
<td>10.61</td>
<td>1.57</td>
<td>2.17</td>
<td>1.62</td>
<td>2.91</td>
<td>0.01</td>
</tr>
<tr>
<td>Account for public spending, the size of the welfare-pie ( z(\xi) )</td>
<td>29.63</td>
<td>29.20</td>
<td>28.87</td>
<td>29.18</td>
<td>28.66</td>
<td>-2.48</td>
</tr>
<tr>
<td>Average taxable income—the wealth amount ( W(\xi) )</td>
<td>105.04</td>
<td>109.95</td>
<td>108.86</td>
<td>109.87</td>
<td>107.88</td>
<td>120.46</td>
</tr>
<tr>
<td>Wealth-tax, marginal tax rate ( \tau(\xi) )</td>
<td>28.21%</td>
<td>26.56%</td>
<td>26.52%</td>
<td>26.56%</td>
<td>26.56%</td>
<td>-2.06%</td>
</tr>
</tbody>
</table>
Figure 4. The monetary expectations of social and public organizations are depicted on vertical and horizontal axes respectively. The graph shows the bargaining frontier of primary or basic goods sloping down from left top \( \xi_2 \) toward right bottom \( \xi_1 \). It is the projection of efficient NIT-gross levels \( \xi \) resolving the frontier constraint (5).

Figure 5. The graph represents two different motions for a vote—the higher tax \( \tau = 29.01\% \), marked by the blue horizontal line, and the lowest tax \( \tau = 26.52\% \), marked by vertical dash. On the blue line from the dash line to the left, and to the right, at the cross points with the red frontier, we observe conflicting expectations of voters. To the left—the lower basic but higher public, and to the right—the higher basic but lower public goods. Thus, the higher tax motion 29.01% for a vote \( \tau \) cannot pass by unanimous consent, Observation 5.